

A Simple Coupled-Mode Analysis Method for Multiple-Core Optical Fiber and Coupled Dielectric Waveguide Structures

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Abstract—A simple method is proposed for the coupled-mode analysis of multiple-core optical fiber structures and coupled dielectric waveguide structures. The coupling coefficients between two adjacent cores are first estimated based on the point matching of boundary conditions on the surface of the two cores. The coupled-mode fields of the multiple-core structures are then approximated by using the fields of the two adjacent cores. Parameters calculated with this procedure are compared with those obtained from a more rigorous analysis.

I. INTRODUCTION

OPTICAL fibers with multiple-core structures will have great importance in the future application of high-density optical transmission lines [1]–[3]. Multiple-waveguide optical coupler structures have been reported that make use of the sharper transfer characteristics of such devices [4], [5]. Coupled dielectric waveguide structures are also expected to be used in millimeter-wave applications. In the design of these structures, it is desirable to estimate evanescent coupling properties between cores by using simple coupled-mode expressions.

We propose in this paper a simple coupled-mode analysis method for describing the coupling properties of multiple-core structures by neglecting coupling effects between nonadjacent cores at first. The propagation characteristics of an isolated dual-core fiber are examined by using the point matching of boundary conditions [6]–[8] to estimate the total coupled-mode fields of multiple-core structures. This procedure corresponds to Hückel's approximation in the analysis of molecular orbitals governed by Schrödinger's equation [9], [10] and is applied to dielectric waveguide problems for the first time, to the authors' knowledge, in this paper.

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II. COUPLED-MODE EQUATIONS FOR DUAL-CORE STRUCTURES

Fig. 1 shows typical multiple-core structures, i.e., (a) n circularly distributed cores [7], (b) n circularly distributed cores and a central core [1], [3], [8], and (c) n linearly distributed cores [3]–[5]. We classify these structures as type 1, type 2, and type 3, respectively. The core separation of these structures is denoted by d or D , where D for type 2 is defined by $D = (d/2)/\sin(2\pi/n)$. Each of these cores is of the same circular shape and has the same dielectric constant ϵ_1 and the same radius a . The cladding has the dielectric constant ϵ_c . We assume that all dielectric regions are lossless, isotropic, and uniform along the propagation axis (z axis).

The approximation method proposed in this paper is based partly on a conventional coupled-mode analysis and partly on coupled transmission line equations whose matrix elements are identical to coupling coefficients.

We first consider the following basic coupled-mode equations for the dual-core structure shown in Fig. 2, whose propagation constants of the individual cores are equally β_0 :

$$-j\beta \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = -j \begin{bmatrix} \beta_0 & C(d) \\ C(d) & \beta_0 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \quad (1)$$

where A_i is the amplitude of the transverse electric field of the i th core ($i=1,2$), $C(d)$ is the coupling coefficient between the cores whose separation is d , and β is the propagation constant of the coupled mode.

Equation (1) can be easily solved as shown below:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix} \quad (2a)$$

$$\beta = \beta^\pm(d) = \beta_0 \pm C(d). \quad (2b)$$

In the scalar wave approximation, the solutions of (2a) and (2b) define the field pattern shown in Fig. 3(a) and (b), respectively.

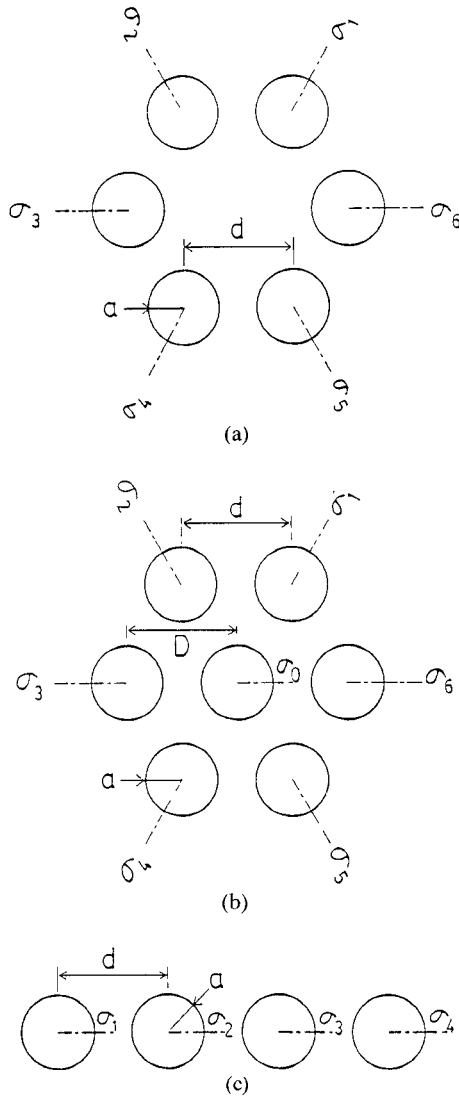


Fig. 1. Multiple-core structures to be considered in this analysis. (a) Circularly distributed cores. (b) Circularly distributed cores and a central core. (c) Linearly distributed cores.

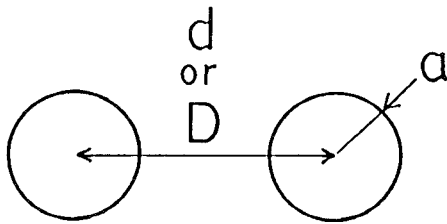


Fig. 2. Dual-core structure used for the estimation of the coupling coefficients between adjacent cores.

The propagation constants, $\beta^+(d)$ and $\beta^-(d)$, can also be calculated from the results of numerical analyses, such as the point-matching method [6]–[8]. Therefore, the coupling coefficient [5] $C(d)$ can be derived from these propagation constants obtained from the point-matching analysis. The result is

$$C(d) = (\beta^+(d) - \beta^-(d))/2 = \Delta\beta(d)/2. \quad (3)$$

In the formulation of coupled-mode equations for the multiple-core structures shown in Fig. 1, we neglect the

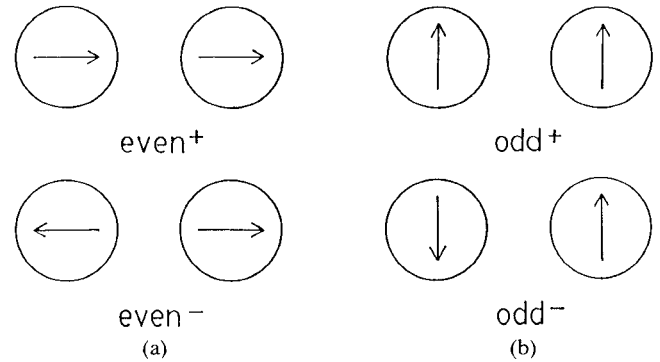


Fig. 3. Simplified field patterns of a dual-core structure. (a) Even⁺ mode and odd⁺ mode. (b) Even⁻ mode and odd⁻ mode.

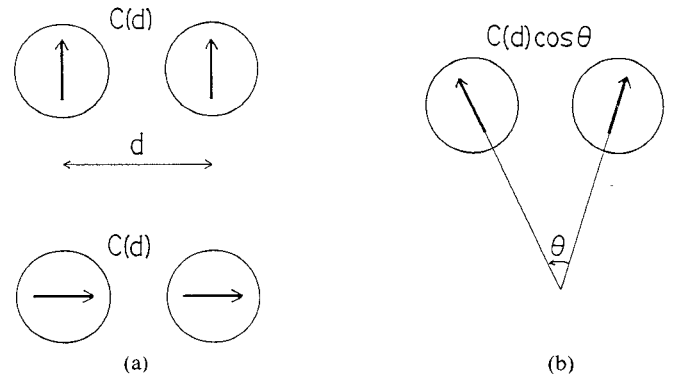


Fig. 4. Coupling coefficient between two field vectors taking into account the angle made by these two field vectors. (a) Coupling coefficient $C(d)$ between two parallel vectors. (b) Field coupling coefficient $C(d)\cos\theta$ between two field vectors with the angle θ defined by $C(d)$.

coupling coefficients of nonadjacent cores because these are considered to be very small in the first-order approximation.

Coupling coefficients between two adjacent cores are obtained from (3), i.e., from the differences of the propagation constants of a dual-core structure whose core separation equals that of the adjacent two cores of a multiple-core structure. Since the orientation of the transverse electric field vectors is not always parallel in these structures, we must take into account the angle θ made by these vectors (Fig. 4). Therefore, the field coupling coefficient $C(d, \theta)$ between these vectors can be defined here as

$$C(d, \theta) = C(d) \cdot \cos\theta = \frac{\Delta\beta(d)}{2} \cos\theta. \quad (4)$$

The field coupling coefficients are employed in the next section to solve multiple-core coupling problems.

III. COUPLED-MODE EQUATIONS FOR MULTIPLE-CORE STRUCTURES

Type 1: n Circularly Distributed Cores

We denote the components of the transverse electric field vector of the i th core A_i by A_i^e or A_i^o depending on their orientation, parallel or orthogonal, to the σ_i axis ($i=1, \dots, n$) as shown in Fig. 1, respectively. Fig. 5 shows

the relation between A_i^e , A_i^o , and A_i . Simplified coupled-mode equations for A_i^e and A_i^o are then written as

$$-j\beta_m \begin{bmatrix} A_1^e \\ \vdots \\ A_n^e \\ A_1^o \\ \vdots \\ A_n^o \end{bmatrix} = -j \begin{bmatrix} \beta_0 & C_1 & & & C_1 \\ C_1 & \ddots & & & 0 \\ & \ddots & \ddots & & C_1 \\ & & 0 & \ddots & \beta_0 \\ C_1 & & C_1 & \beta_0 & \\ 0 & C_2 & & C_2 & \\ C_2 & & & 0 & \\ & & \ddots & & C_2 \\ C_2 & & C_2 & 0 & \end{bmatrix} \begin{bmatrix} A_1^e \\ \vdots \\ A_n^e \\ A_1^o \\ \vdots \\ A_n^o \end{bmatrix} \quad (5a)$$

where

$$C_1 = C\left(d, \frac{2\pi}{n}\right) = \frac{\Delta\beta(d)}{2} \cos \frac{2\pi}{n} \quad (5b)$$

$$C_2 = C\left(d, \frac{2\pi}{n} - \frac{\pi}{2}\right) = \frac{\Delta\beta(d)}{2} \sin \frac{2\pi}{n}. \quad (5c)$$

The propagation constant of mode m is obtained by solving for the eigenvalues of (5a):

$$\beta_m = \beta_0 + \Delta\beta(d) \cdot \cos\left\{\frac{2\pi}{n}(m-1)\right\}. \quad (6a)$$

The column vectors of amplitude coefficients $[A_1^e, \dots, A_n^e, A_1^o, \dots, A_n^o]^T$ of the even and the odd mode, which we denote by A_m^e, A_m^o , are doubly degenerate, and given by

$$A_m^e = \frac{1}{\sqrt{n}} \begin{bmatrix} \cos\left(\frac{2\pi}{n}m \cdot 1\right) \\ \vdots \\ \cos\left(\frac{2\pi}{n}mn\right) \\ -\sin\left(\frac{2\pi}{n}m \cdot 1\right) \\ \vdots \\ -\sin\left(\frac{2\pi}{n}mn\right) \end{bmatrix} \quad A_m^o = \frac{1}{\sqrt{n}} \begin{bmatrix} \sin\left(\frac{2\pi}{n}m \cdot 1\right) \\ \vdots \\ \sin\left(\frac{2\pi}{n}mn\right) \\ \cos\left(\frac{2\pi}{n}m \cdot 1\right) \\ \vdots \\ \cos\left(\frac{2\pi}{n}mn\right) \end{bmatrix} \quad (6b)$$

Type 2: n Circularly Distributed Cores and a Central Core

From the results given for type 1, eigenmodes can be considered as the coupled modes of mode m for circularly distributed cores and the propagation mode of the central core. Their amplitude coefficients are denoted by A_{ms}^p and A_{m0}^p , respectively ($m = 0, 1, \dots, n$; $p = e, o$). Therefore,

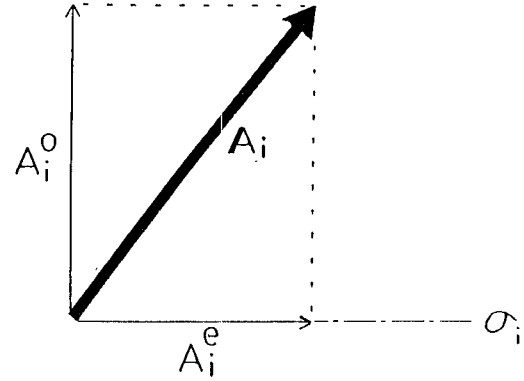


Fig. 5. The relation between the two components, A_i^e and A_i^o , of the vector A_i .

coupled-mode equations for this case can be written as

$$-j\beta_m \begin{bmatrix} A_{ms}^p \\ A_{m0}^p \end{bmatrix} = -j \begin{bmatrix} \beta_s & C_{cs} \\ C_{cs} & \beta_0 \end{bmatrix} \begin{bmatrix} A_{ms}^p \\ A_{m0}^p \end{bmatrix} \quad (7a)$$

where β_s is the propagation constant of mode m of the circularly distributed cores (eq. (6a)) and C_{cs} is the field coupling coefficient between a propagation mode for circularly distributed cores (mode m in type 1) and a propagation mode for a central core. The field coupling coefficient in the case of the even mode can be expressed as

$$C_{cs} = \sum_{i=1}^n \left\{ C\left(D, \frac{2\pi}{m}i\right) \cdot \frac{1}{\sqrt{m}} \cos\left(\frac{2\pi}{n}mi\right) + C\left(D, \frac{2\pi}{n}i + \frac{\pi}{2}\right) \cdot \frac{-1}{\sqrt{n}} \sin\left(\frac{2\pi}{n}mi\right) \right\} \\ = \begin{cases} \sqrt{n} \frac{\Delta\beta(D)}{2} & (m=1) \\ 0 & (m \neq 1). \end{cases} \quad (7b)$$

This result is found to be the same as the case of the odd mode after a similar calculation. Finally, the solutions of

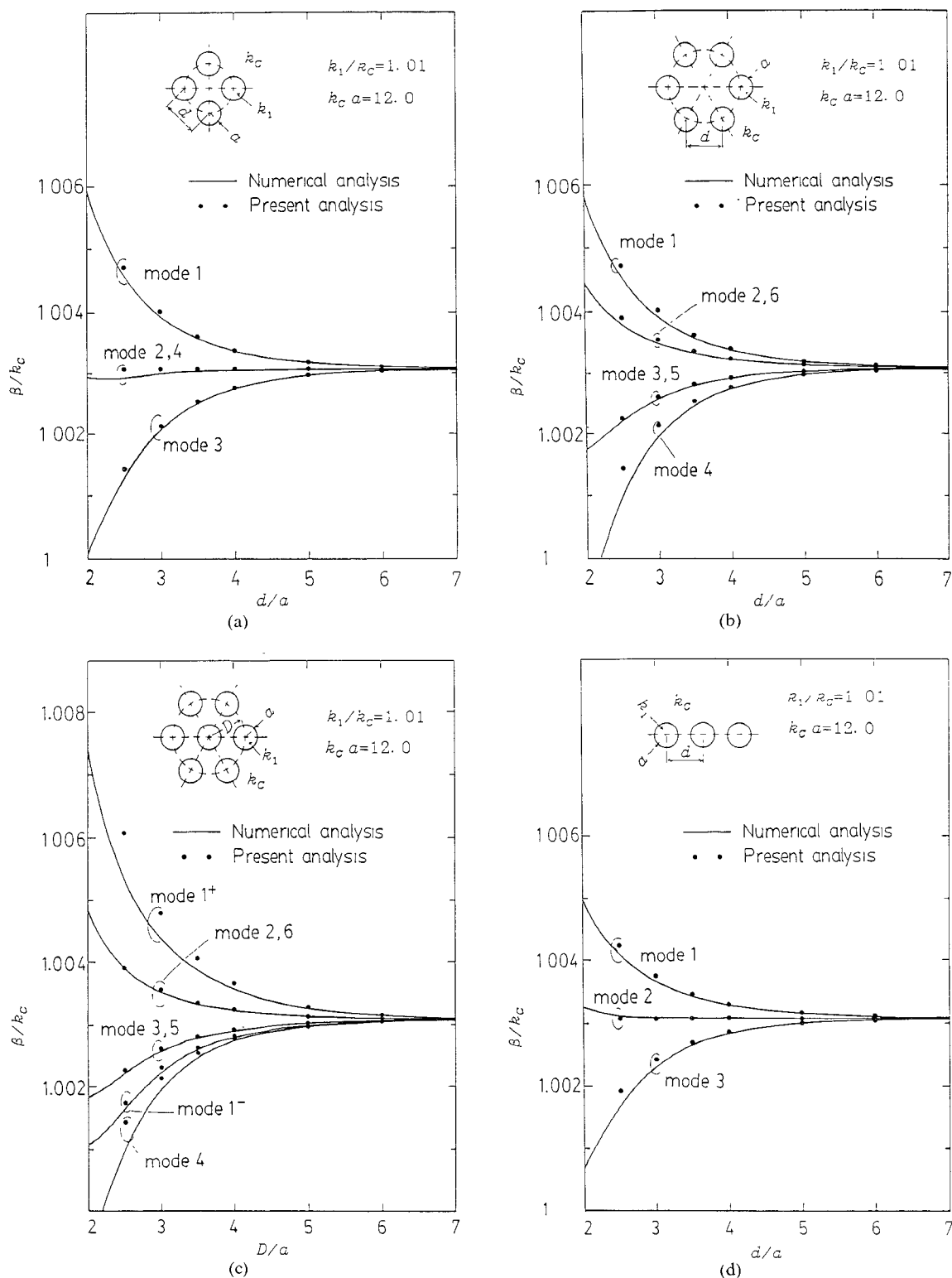


Fig. 6. Comparison of the dependence of β/k_c on the core separation obtained from the present method and that from a rigorous numerical analysis [8], [9]. (a) Circularly distributed cores ($n=4$). (b) Circularly distributed cores ($n=6$). (c) Circularly distributed cores and a central core ($n=6$). (d) Linearly distributed cores ($n=3$).

(7a) can be written as

$$\beta_m = \begin{cases} \beta_0 + \frac{\Delta\beta(d) \pm \sqrt{\Delta\beta(d)^2 + n \cdot \Delta\beta(D)^2}}{2} & (m=1) \\ \beta_s = \beta_0 + \Delta\beta(d) \cdot \cos\left\{\frac{2\pi}{n}(m-1)\right\} & (m \neq 1) \end{cases} \quad (8a)$$

$$A_{ms}^p = 1 \quad (m=1, 2, \dots, n) \quad (8b)$$

$$A_{m0}^p = \begin{cases} \frac{1}{\sqrt{n}} \left\{ -\frac{\Delta\beta(d)}{\Delta\beta(D)} \pm \sqrt{n + \frac{\Delta\beta(d)^2}{\Delta\beta(D)^2}} \right\} & (m=1) \\ 0 & (m \neq 1) \end{cases} \quad (8c)$$

There are two modes for the mode number $m=1$. These modes are distinguished by the superscripts $+$ and $-$ with the relation $\beta_1^+ > \beta_1^-$ [8].

Type 3: n Linearly Distributed Cores

Using notation and symbols similar to those for types 1 and 2, coupled-mode equations can be written as

$$-j\beta_m \begin{bmatrix} A_1^p \\ \vdots \\ A_n^p \end{bmatrix} = -j \begin{bmatrix} \beta_0 & \frac{\Delta\beta(d)}{2} & & 0 \\ \frac{\Delta\beta(d)}{2} & & & \\ & \ddots & \ddots & \\ 0 & & \frac{\Delta\beta(d)}{2} & \beta_0 \end{bmatrix} \begin{bmatrix} A_1^p \\ \vdots \\ A_n^p \end{bmatrix} \quad (9)$$

The solution of (9) is also given in a fashion similar to those of the preceding types as

$$\beta_m = \beta_0 + \Delta\beta(d) \cdot \cos \frac{m\pi}{n+1} \quad (10a)$$

$$A_m^p = \begin{bmatrix} A_1^p \\ \vdots \\ A_n^p \end{bmatrix} = \begin{bmatrix} \sin \frac{m \cdot 1 \cdot \pi}{n+1} \\ \vdots \\ \sin \frac{mn\pi}{n+1} \end{bmatrix} \quad (10b)$$

IV. RESULTS OF MODAL ANALYSIS

In this section, we describe results of the modal analysis given in the previous section. Fig. 6 shows a comparison of the propagation constants obtained by the present method

and those by previous numerical analyses [7], [8] where the wavenumber of the core and the cladding, k_1 and k_c , are defined by $\omega\sqrt{\epsilon_1\mu_0}$ and $\omega\sqrt{\epsilon_c\mu_0}$, respectively. As the separation between cores is decreased, the effect of coupling between nonadjacent cores naturally become strong and the accuracy of the results of the present method is slightly reduced.

The state of degeneracy, the order of propagation modes, and field patterns [8, fig. 5] can also be obtained from (6), (8), and (10), regardless of the size of core separation. These results are consistent with the LP-mode notations given by Gloge [11].

V. RESULTS OF COUPLING-FIELD ANALYSIS

In this section, we analyze the coupling properties of the HE_{11} mode in multiple-core structures using (6), (8), and (10) to see how the power incident to one core is transferred to other cores. From the preceding equations, it is easily found that the polarization of coupling fields into other cores is parallel to the incident polarization. Therefore, we do not have to consider the orientation of the field but the amplitude. The coupling field is expressed with the superposition of the eigenmodes given by (6), (8), and (10).

The amplitudes of individual eigenmodes are determined by the excitation field at $z=0$. When one of the cores is excited at $z=0$ by a unit electric field, $e_i(z)$, the complex amplitude of the electric field of the i th core at a distance z is derived as follows.

Type 1: n Circularly Distributed Cores

We only have to consider the n th core excitation [8]. In this case, $e_i(z)$ ($i=1, 2, \dots, n$) is given as

$$e_i(z) = \frac{1}{n} e^{-j\beta_0 z} \sum_{m=1}^n \cos\left(\frac{2\pi}{n}mi\right) \cdot e^{-j\Delta\beta(d)\cos(2\pi m/n) \cdot z} \quad (11)$$

Type 2: n Circularly Distributed Cores and a Central Core

In the case of n th core excitation, $e_i(z)$ is expressed as

$$e_i(z) = \frac{1}{n} e^{-j\beta_0 z} \left\{ \frac{R \cos \frac{RZ}{2} - j\Delta\beta(d) \cdot \sin \frac{RZ}{2}}{R} e^{-j\Delta\beta(d)/2Z} + \sum_{m=1}^{n-1} \cos\left(\frac{2\pi}{n}mi\right) \cdot e^{-j\Delta\beta(d) \cos(2\pi m/n) \cdot z} \right\} \quad (i=1, 2, \dots, n) \quad (12a)$$

$$e_0(z) = -j \frac{\Delta\beta(D)}{2} e^{-j\beta_0 z} \cdot \sin\left(\frac{RZ}{2}\right) \cdot e^{-j\Delta\beta(d)/2Z} \quad (12b)$$

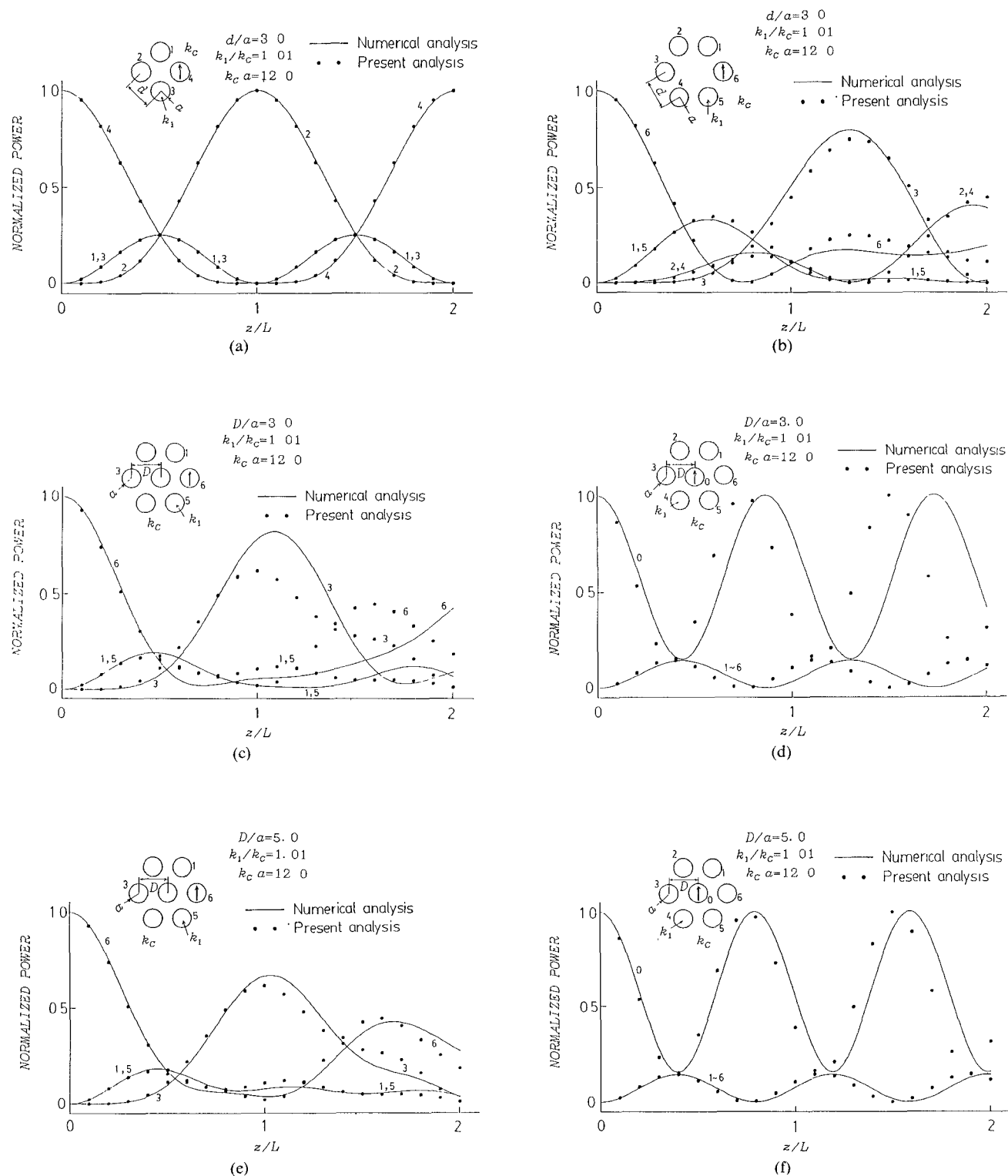


Fig. 7. Comparison of the coupling characteristics obtained from the present method and those from a rigorous numerical analysis [9]. (a) circularly distributed cores ($n=4$). (b) Circularly distributed cores ($n=6$). (c) Circularly distributed cores and a central core ($n=6$, the sixth core excitation). (d) Circularly distributed cores and a central core ($n=6$, the central core excitation). (e) Circularly distributed cores and a central core for a large core separation ($n=6$, the sixth core excitation). (f) Circularly distributed cores and a central core for a large core separation ($n=6$, the central core excitation) (Continued).

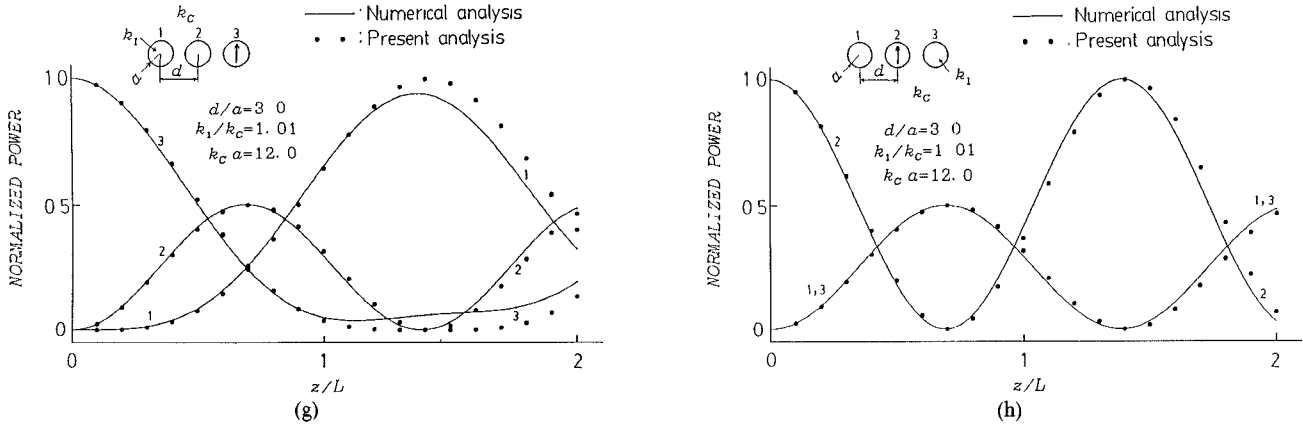


Fig. 7. (Continued) (g) Linearly distributed cores ($n=3$, the second core excitation). (h) Linearly distributed cores ($n=3$, the third core excitation).

In the case of central (0th) core excitation, $e_i(z)$ is given by

$$e_i(z) = -j \frac{\Delta\beta(D)}{R} e^{-j\beta_0 z} \sin\left(\frac{RZ}{2}\right) e^{-j\Delta\beta(d)/2Z} \quad (i=1,2,\dots,n) \quad (13a)$$

$$e_0(z) = e^{-j\beta_0 z} \frac{R \cos \frac{RZ}{2} + j\Delta\beta(d) \sin \frac{RZ}{2}}{R} \quad (13b)$$

where R is defined as

$$R = \sqrt{n \cdot \Delta\beta(D)^2 + \Delta\beta(d)^2}. \quad (14)$$

Type 3: n Linearly Distributed Cores

In the case of l th core excitation, $e_i(z)$ is given as

$$e_i(z) = \frac{2}{n+1} e^{-j\beta_0 z} \sum_{m=1}^n \sin \frac{ml\pi}{n+1} \cdot \sin\left(\frac{mi\pi}{n+1}\right) \cdot e^{-j\Delta\beta(d) \cos\{m\pi/(n+1)\}Z}. \quad (15)$$

The normalized power $P_i(z)$ in the individual cores is naturally obtained from $e_i(z)$ as

$$P_i(z) = e_i(z) e_i^*(z). \quad (16)$$

Fig. 7 shows the normalized power in each core versus the normalized propagation length. The length is normalized by the beat length L for a dual-core system which has the core separation d (in the case of type 1 and type 3) or D (in the case of type 2). The values obtained by the present method are compared with those by a previous numerical analysis as shown in Fig. 7 [8]. Even when the core separation d or D is small (d/a or D/a is equal to 3), good agreement between these values is seen except for type 2. When the core separation D becomes large in type 2, good agreement is also seen between these values.

VI. DISCUSSION

1) We have proposed in this paper a simple coupled-mode analysis (scalar wave analysis) for coupled dielectric waveguide structures such as optical fibers with multiple cores. It has been found from the foregoing results that

this method is valid and useful for providing approximate solutions to these types of multiple-waveguide problems.

2) We have been able to express the coupled modes of the multiple-core structures in the explicit and simple form given by (6), (8), and (10) using this approximation method. The state of degeneracy and the order of propagation modes given by (6), (8), and (10) are consistent with the results of point-matching analysis regardless of the size of core separation. In particular eigenmodes in type 1 and type 2 given by (6) and (8) have a form identical to those obtained from group theory [7]–[10].

The effect of the central core can be explained from the content of (7) and (8) of type 2. The field coupling coefficient between the mode of type 1 and the mode of the central core has been found to be zero except for the case $m=1$.

3) Fig. 6 shows that if the core separation becomes larger, the effect of the coupling between nonadjacent cores becomes weaker, so that results with the present approximation method become more accurate.

4) It can be stated from the results shown in Fig. 7 that the accuracy of the coupling field calculation in type 1 and type 3 is better than that in type 2 when the core separation is relatively small. If the core separation becomes larger, however, the accuracy of coupling field calculation in type 2 is improved.

VII. CONCLUSION

The proposed approximation method has been found to be a simple, efficient, and practical tool for the coupled-mode characterization of complicated multiple-core fiber structures or multiple dielectric waveguide structures. It should be pointed out, however, as noticed in some of our results, that this method might not work well for very small separations between dielectric waveguides.

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